LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **COMPUTER APPLICAION**

FIRST SEMESTER – **NOVEMBER 2013**

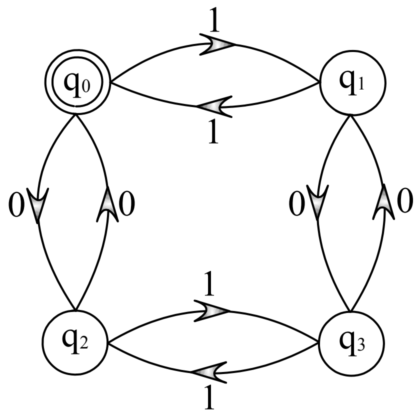
MT 1902 - MATHEMATICS FOR COMPUTER APPLICATIONS

Date : 13/11/2013 Dept. No. Max. : 100 Marks
Time : 1:00 - 4:00

SECTION A

Answer ALL the questions. **(10 x 2 = 20)**

1. Define Lattice homomorphism between two lattices.
2. With usual notations prove that (i) $a * a = a$(ii)$a * b = b * a$.
3. Define context free grammar.
4. For a DFA $M = (\{q_0, q_1, q_2, q_3\}, \{0,1\}, \delta, q_0, \{q_0\})$,



Show that the string 011011 is in $L(M)$.

5. Define a Phrase – Structure grammar.
6. State the Pigeon hole principle.
7. Draw the Hasse diagram for the divisors of 32.
8. Define a bipartite graph.
9. If G is a group then prove that $ab = ac$ implies that $b = c$ for all $a, b, c \in G$.
10. Check whether the relation ">"defined on $N = \{1, 2, 3, ...\}$ is a total order relation.

SECTION B

Answer ALL the questions. **(5 x 8 = 40)**

11. (a) Prove that the complement a' of any element 'a' of a Boolean algebra is uniquely determined. Prove also that the map $a \to a'$ is an anti – automorphism of period $\leq$ 2 and $a \to a'$ satisfies $(a \vee b)' = a' \wedge b'$, $(a \wedge b)' = a' \vee b'$, $a'' = a$.

(or)

(b) Discuss 'negation' and explain a method of constructing the truth table for $P \vee \neg Q$ and $(P \vee Q) \vee \neg P$

12. (a) Write a short note on principal conjunctive normal form and construct an equivalent formulafor $\neg(P \vee Q) \rightleftarrows (P \wedge Q)$.

(or)

(b) For a grammar $G = (\{S, Z, A, B\}, \{a, b\}, P, S)$ where P consists of the following production:

1. $S \rightarrow aSA$ 4. $Z \rightarrow bB$ 7. $bB \rightarrow bb$

2. $S \rightarrow aZA$ 5. $BA \rightarrow AB$ 8. $bA \rightarrow ba$

3. $Z \rightarrow bZB$ 6. $AB \rightarrow Ab$ 9. $aA \rightarrow aa$

Then show that $L(G) = \{a^n b^m a^n b^m / n, m \geq 1\}$.

13. (a) Define a Regular Expression.

(b) Explain the equivalence of deterministic finite automata and regular expressions.

(or)

(c) For a grammer $G = (V, T, P, S)$ where $V = \{S\}, T = \{a, b\}$ and $P = \{S \rightarrow aSb, S \rightarrow ab\}$, then show that $L(G) = \{a^n b^n / n \geq 1\}$.

(d) Show that the relation R = $\{(a, b) / a - b = k\, m$ for some fixed integer m and $a, b, k \in Z\}$ is an equivalence relation.

14. (a) Show that $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$.

(b) Prove that (i) $A - B = A \cap B'$.(ii) $(A \cup B) \cup C = A \cup (B \cup C)$.

(or)

(c) If f is a mapping from X onto Y then for each subset $A \subseteq Y$, prove that $f\left(f^{-1}(A)\right) = A$.

(d) Prove that the inverse of a one to one and onto function is one to one and onto.

15. (a) Prove that there is a one- to-one correspondence between any two left cosets of H in G.

(or)

(b) (i) If G is a graph in which the degree of every vertex is atlest two, then prove that G contains a cycle.

(ii) Prove that the kernel of a homomorphism g from a group $\langle G, * \rangle$ to $\langle H, \Delta \rangle$ is a subgroupof $\langle G, * \rangle$. (4+4)

Section C

Answer any TWO questions. (2 x 20 = 40)

16. (a) Let G be (p,q)graph, then prove that the following statements are equivalent:

(i) G is a tree. (ii) Every two vertices of G are joined by a unique path (iii) G is connected and $p = q+1$ (iv) G is acyclic and $p = q+1$.

(b) Let H be a subgroup of G. Then prove that any two left cosets of H in G are either identical or have no element in common. (14+6)

17. (a) State and prove pumping lemma for regular sets.

(b) List any four applications of pumping lemma.

(c)(i) State the inclusion and exclusion principle.

(ii).Find the number of students at a college taking at least one of the languages French, German and Russian, given the following data: 65 study French; 20 study French and German ; 45 study German; 25 study French and Russian; 42 study Russian; 15 study German and Russian; 8 study all three languages.

(10+4+6)

18. (a) Show that in a graph G, any u – v walk contains a u – v path.

(b) Prove that a closed walk of odd length contains a cycle.

(c) State and prove Lagrange theorem. (4 + 4 + 12)